Hyde High School

Numeracy Booklet

Introduction

What is the purpose of this booklet?

This booklet has been produced to give guidance to pupils and parents on how certain common Numeracy topics are taught during maths lessons at Hyde. Staff from all departments have access to a copy of the booklet. It is hoped that using a consistent approach across all subjects will make it easier for pupils to progress.

How can it be used?

Read through the booklet one section at a time and then try the questions that are set at the end of most sections, checking your answers with those given at the end of the booklet. You can also talk to your child as you go through, asking them questions about the various topics. For example, asking them to describe a parallelogram, or what a negative number multiplied by another negative number gives.

If you are helping your child with their homework, you can refer to the booklet to see what methods are being taught in school. Simply look up the relevant page for a step by step guide and useful examples.

Any word underlined is a link to a video demonstration of the method(s) involved in that section of the booklet

This booklet includes skills not only useful in their maths lessons, but also in other subjects across the curriculum and in general outside of school.

For help with maths topics not found in this booklet, pupils should refer to their class work or ask their teacher for help.

Why is more than one method shown?

In some cases the method used will be dependent on the level of difficulty of the question, whether or not a calculator is permitted or simply which method the pupil themselves prefers.

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1. Mental Methods

Addition

Example 1: 54 + 27 Example 2: 126 + 54

Method 1: Add tens and units separately, then add together

1) 50 + 20 = 70   4 + 7 = 11   70 + 11 = **81**
2) 100 + 0 = 100   20 + 50 = 70   6 + 4 = 10   100 + 70 + 10 = **180**

Method 2: Split the second number, add the tens then add the units

1) 54 + 20 = 74   74 + 7 = **81**
2) 126 + 50 = 176   176 + 4 = **180**

Method 3: Round up to the next 10, then subtract the difference

1) 27 rounds up to 30
   54 + 30 = 84
   84 – 3 = **81**

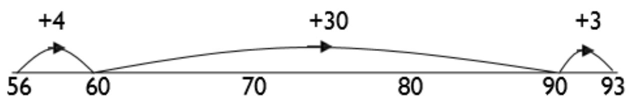
2) 54 rounds up to 60
   126 + 60 = 186
   186 – 6 = **180**

Subtraction

Example 1: 93 – 56 Example 2: 154 – 143

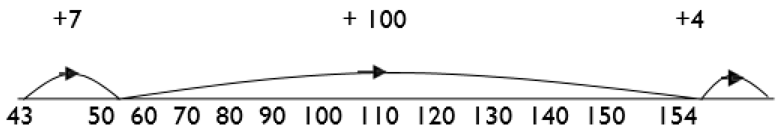
Method 1: Count on

1) Count on from 56 until you reach 93



Answer = **37**

2) Count on from 43 until you reach 154



Answer = **111**

Method 2: Break up the number being subtracted

1) 93 – 50 = 43
43 – 6 = **37**

154 – 40 = 114
114 – 3 = **111**

Multiplication

It is essential that pupils know all of the times tables from 1x 1 up to 10 x 10. These are shown below. Practice can take place on Dr. Frost and should be done daily until they are learned to memory.

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example 1: 39 x 6

Example 2: 27 x 12

Method 1: Multiply by tens, multiply by units. Add together

1) 30 x 6 = 180
9 x 6 = 54
180 + 54 = **234**

2) 27 x 10 = 270
27 x 2 = 54
270 + 54 = **324**

Method 2: Round the multiplier up to the next ten, subtract the multiples

1) 40 x 6 = 240
1 x 6 = 6
240 – 6 = **234**

2) 30 x 12 = 360
3 x 12 = 36
360 – 36 = **324**

2. Written Methods

Addition

Example 1: 534 + 2678

Example 2: 642 + 249

Place the digits in the correct place value with the digits lined up under each other. Begin adding from the units column (right hand side).

Show any carrying in the next column.

Th	H	T	U
1	1	1	
	5	3	4
2	6	7	8
3	**2**	**1**	**2**

Th	H	T	U
		1	
	6	4	2
	2	4	9
	8	**9**	**1**

Subtraction

Example 1: 7689 – 749

Th	H	T	U
6			
~~7~~	16	8	9
	7	4	9
6	9	4	0

Example 2: 3176 – 1482

Th	H	T	U
2	10		
~~3~~	~~1~~	17	6
1	4	8	2
1	6	9	4

Addition of Decimals

Example 1: 53.4 + 26.78

Example: 18.44 + 23. 76

Begin lining up the decimal point to use as a guide. Place each digit in the correct "place value" columns with the numbers under each other. Start adding from the most right column.

T	U	.	t	h
5	$^{1}3$	.	4	
2	6	.	7	8
7	**9**	**.**	**1**	**8**

T	U	.	t	h
$^{1}1$	$^{1}8$	.	$^{1}4$	4
2	3	.	7	6
4	**2**	**.**	**2**	**0**

Subtraction of Decimals

Example 1: 78.9 – 7.49

Example 2: 18.31 – 11.04

Begin lining up the decimal points, use this as your guide. Place the digits in the correct columns with the numbers under each other. Begin subtracting from the furthest column on the right.

If the number on the row above is smaller than the number on the bottom, regroup a ten from the column to the left (seen in red on the examples)

Fill in any gaps with zeros.

T	U	.	t	h
7	8	.	$^{8}\cancel{9}$	$^{1}0$
	7	.	4	9
7	**1**	**.**	**4**	**1**

T	U	.	t	h
1	8	.	$^{2}\cancel{3}$	$^{1}1$
1	1	.	0	4
	7	**.**	**2**	**7**

Multiplication

Example 1: 56 x 34

Example 2: 124 x 7

Method 1 – Grid Method

Split the numbers into their place value parts – units, tens hundreds etc. Write one number along the top and one number down the left hand side. Multiply in each section of the grid until it is complete. Add the numbers inside the grid to calculate the final answer.

	50	6
30	1500	180
4	200	24

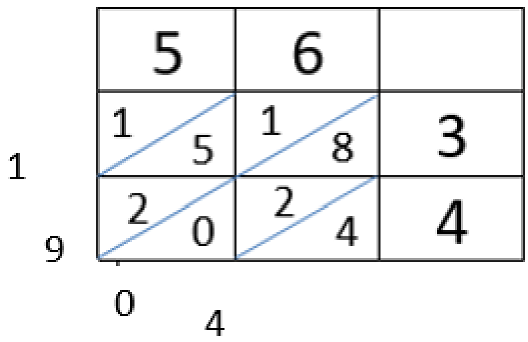
1500 + 180 + 200 + 24 = **1904**

	100	20	4
7	700	140	28

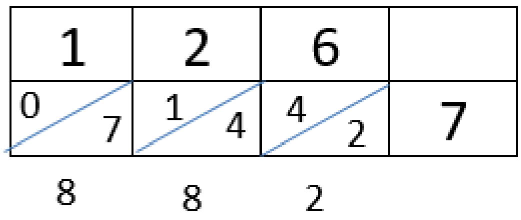
700 + 140 + 28 = **868**

Method 2 – Napier's Bones

Write one of the numbers across the top and the other number down the right hand side. Multiply each single digit by each other single digit. Add the digits right to left.



Answer = **1904**



Answer = **882**

Division

Example 1: 980 ÷ 4

Example 2: 768 ÷ 12

Method 1: Bus Stop Method

Divide each digit by the divisor (number on the outside). Write the whole number part on top of the "bus stop" and carry any remainders over to the next digit. The answer can be found on top of the bus stop method

$$\begin{array}{r|l} & 2\ \ 4\ \ 5 \\ \hline 4 & 9\ \ {}^{1}8\ \ {}^{2}0 \end{array}$$

$$\begin{array}{r|l} & \ \ \ \ 6\ \ 4 \\ \hline 12 & 7\ \ {}^{7}6\ \ {}^{4}8 \end{array}$$

Method 2: Chunking

This method uses addition to calculate division. We add multiples of our divisor (what we are dividing by) until we reach the original number until we reach zero. We will use 1, 2, 5, 10 and 100 as our multiples to keep things straight forward.

We then add up the multiples to calculate the final answer

X	4	Total
100	400	400
100	400	800
10	40	840
10	40	880
10	40	920
10	40	960
5	20	980
245		

X	12	Total
10	120	120
10	120	240
10	120	360
10	120	480
10	120	600
10	120	720
2	24	744
2	24	768
64		

3. Number Properties

Odd and Even

Odd	Even
Any number ending in 1, 3, 5, 7, 9 This is any number not in the 2 timestables Examples 673 (ends in a 3) 232125 (ends in a 5) 999431 (ends in a 1)	Any number ending in 2, 4 6, 8, 0 This is any number in the 2 timestables Examples 34212 (ends in a 2) 543980 (ends in a 0) 432456 (ends in a 6)

Square Numbers

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

This is a list of the first 15 square numbers – you need to learn this list!

Square numbers are created by multiplying a number by itself

1 x 1 = **1**   2 x 2 = **4**   3 x 3 = **9**   4 x 4 = **16**

They are called square numbers because they represent the area of a square

Cube Numbers

1, 8, 27, 64, 125

This is a list of the first 5 cube numbers – you need to learn these!

Cube numbers are created by multiplying a number by itself and itself again

1 x 1 x 1 = **1**   2 x 2 x 2 = **8**   3 x 3 x 3 = **27**

They are called cube numbers because they represent the volume of a cube

Prime Numbers

2, 3, 5, 7, 11, 13, 17,

This is a list of the first 7 prime numbers.

A prime number has exactly 2 factors: 1 and itself. **1 is not a prime number**

Factors and Multiples

Factors	Multiples
A number which can divide another number to produce a whole answer Factors are **smaller or equal** to the original number Example Factors of 6: 1, 2, 3, 6 Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 Factors of 39: 1, 3, 13, 39	A number which has been produced by multiplying 2 whole numbers Multiples are **larger or equal** to the original number Example Multiples of 6: 6, 12, 18, 24 … Multiples of 24: 24, 48, 72, 96 … Multiples of 39: 39, 78, 117, 156 …

4. Place Value

Thousands (Th)	Hundreds (H)	Tens (T)	Units (U)	.	Tenths (t)	Hundredths (h)	Thousandths (th)
1000	100	10	1		0.1	0.01	0.001
					$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Each column is 10 times bigger than the column to its right

The column tells us the value of each digit

3157

This number has 3 thousands (3000), 1 hundred (100), 5 tens (50) and 7 units (7)

4.235

This number has 4 units (4), 2 tenths (0.2), 3 hundredths (0.03) and 5 thousandths (0.005)

5. Fractions

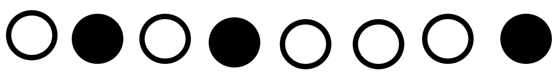
Understanding Fractions

$$\frac{\boldsymbol{numerator}}{\boldsymbol{denominator}}$$

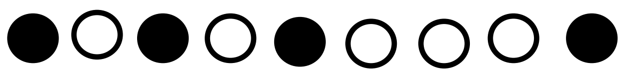
The numerator tells us how many pieces are represented out of the whole group

The denominator tells us how many equal pieces there are in total

What fraction of the circles are black?



There are 8 circles in total, 3 of which are black, so the fraction is written $\frac{3}{8}$



There are 9 circles in total, 4 of which are black, so the fraction is written $\frac{4}{9}$

Equivalent Fractions

Fractions are equivalent if they represent the same proportion

$\frac{1}{2}$ $\frac{2}{4}$ $\frac{4}{8}$

The above tables are all half shaded yet can be represented by fractions with different numbers in them. Hence all fractions above are equivalent

Simplifying Fractions

Fractions can be simplified if the numerator and denominator can be divided by the same number. We do this to make calculations easier.

A fraction is in its simplest form if the numerator and denominator cannot be divided by anything other than 1.

$$\frac{2}{5}\ is\ in\ simplest\ form\ because\ 2\ and\ 5\ only\ have\ 1\ as\ a\ common\ divisor$$

$$\frac{3}{7}\ is\ in\ simplest\ form\ because\ 3and\ 7\ only\ have\ 1\ as\ a\ common\ divisor$$

$$\frac{9}{10}\ is\ in\ simplest\ form\ because\ 9\ and\ 10\ only\ have\ 1\ as\ a\ common\ divisor$$

Example

(a) $$\overset{\div 5}{\underset{\div 5}{\frac{20}{25} = \frac{4}{5}}}$$

(b) $$\overset{\div 8}{\underset{\div 8}{\frac{16}{24} = \frac{2}{3}}}$$

Adding and Subtracting Fractions

When we add or subtract fractions, we need to make sure the fractions have the same denominator. We will use the same method for addition and subtraction, changing only the sign in the question.

Example 1: $\frac{2}{5} + \frac{3}{10}$ Example 2: $\frac{4}{7} - \frac{1}{3}$

Method 1: Equivalent Fractions

Change the fractions in the questions into equivalent fractions so that both denominators are the same.

1) $\frac{2}{5} = \frac{4}{10}$

$$\frac{2}{5} + \frac{3}{10} = \frac{4}{10} + \frac{3}{10} = \frac{4+3}{10} = \mathbf{\frac{7}{10}}$$

2) $\frac{4}{7} = \frac{12}{21}, \frac{1}{3} = \frac{7}{21}$

$$\frac{4}{7} - \frac{1}{3} = \frac{12}{21} - \frac{7}{21} = \frac{12-7}{21} = \mathbf{\frac{5}{21}}$$

Method 2: Cross – Multiplying

Multiply each numerator by the opposite denominator. Multiply the denominators together. Simplify your answer if possible

1) $\frac{2}{5} + \frac{3}{10} = \frac{(2\times10)+(3\times5)}{5\times10} = \frac{20+15}{50} = \frac{35}{50} = \mathbf{\frac{7}{10}}$

2) $\frac{4}{7} - \frac{1}{3} = \frac{(4\times3)-(1\times7)}{7\times3} = \frac{12-7}{21} = \mathbf{\frac{5}{21}}$

Multiplying Fractions

When multiplying fractions, we can multiply numerators and denominators separately. We can simplify our answer before or after our calculation.

Example 1: $\frac{3}{8} \times \frac{7}{12}$

Example 2: $\frac{9}{5} \times \frac{4}{18}$

Method 1: Simplifying after calculating

1) $\frac{3}{8} \times \frac{7}{12} = \frac{3\times7}{8\times12} = \frac{21}{96} = \mathbf{\frac{7}{32}}$

2) $\frac{9}{5} \times \frac{4}{18} = \frac{9\times4}{5\times18} = \frac{36}{90} = \mathbf{\frac{2}{5}}$

Method 2: Simplifying before calculating

1) $\frac{3}{8} \times \frac{7}{12} = \frac{1}{8} \times \frac{7}{4} = \frac{1\times7}{8\times4} = \mathbf{\frac{7}{32}}$

2) $\frac{9}{5} \times \frac{4}{18} = \frac{9}{5} \times \frac{2}{9} = \frac{1}{5} \times \frac{2}{1} = \frac{1\times2}{5\times1} = \mathbf{\frac{2}{5}}$

We can simplify across the fraction. In example 1, we can simplify 3 and 12 as they both divide by 4.

Dividing Fractions

We divide fractions by converting the question into a multiplication. We use the fact that multiplication is the inverse of division, so can invert our divisor

Example 1: $\frac{5}{8} \div \frac{6}{11}$

1) $\frac{5}{8} \div \frac{6}{11} = \frac{5}{8} \times \frac{11}{6} = \frac{5 \times 11}{8 \times 6} = \mathbf{\frac{55}{48}}$

Example 2: $\frac{10}{7} \div \frac{4}{9}$

2) $\frac{10}{7} \div \frac{4}{9} = \frac{10}{7} \times \frac{9}{4} = \frac{10 \times 9}{7 \times 4} = \mathbf{\frac{90}{28}}$

Fractions of Amounts

Example 1: $\frac{1}{5}$ of 80

1) 80 ÷ 5 = 16
   16 x 1 = **16**

Example 2: $\frac{3}{7}$ of 42

2) 42 ÷ 7 = 6
   6 x 3 = **18**

6. Percentages

Percentages represent information out of 100. Percentages allow us to compare scores when they have a different number of events.

Who has done better on a test: scoring 18 out of 30 or 15 out of 20?

Percentages can compare them much more equally than raw scores

Writing Percentages

Begin by writing as a fraction and then converting into a fraction out of 100. The percentage can then be read off.

Examples

24 / 100 = **24%** (we can just read off the top number since the denominator is 100)

17/ 50 = 34 / 100 = **34%** (we convert the original fraction into an equivalent with 100 as the denominator)

12 / 40 = 6 / 20 = 30 / 100 = **30%** (we simplified the fraction to make it easier to convert to a denominator with 100)

Percentages of Amounts

Example 1: 30% of 80

Example 2: 22% of 240

Method 1: Multiplying Fractions

1) $30\% = \frac{30}{100} = \frac{3}{10}$

$\frac{3}{10} \times 80 = \mathbf{24}$

2) $22\% = \frac{22}{100} = \frac{11}{50}$

$\frac{11}{50} \times 240 = 52.8$

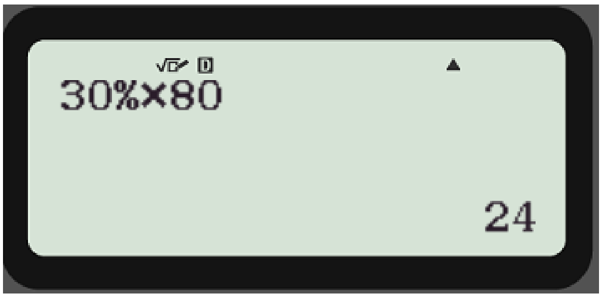
Method 2: 1, 2, 5 and 10

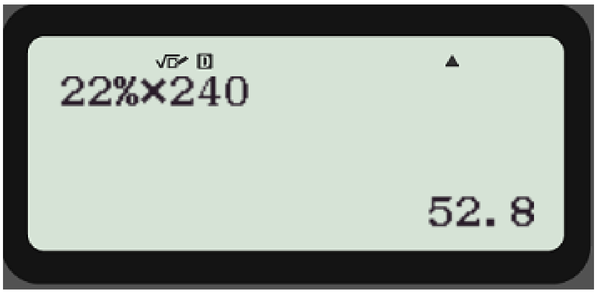
In this method, we calculate 1%, 2%, 5% and 10% of the number

1) 1% = 0.8, 2% = 1.6, 5% = 4, 10% = 8
   30% = 3 x 10% = 3 x 8 = **24**

2) 1% = 2.4, 2% = 4.8, 5% = 12, 10% = 24
   22% = 10% + 10% + 2% = 24 + 24 + 4.8 = **52.8**

Method 3: Calculator





7. Ratio and Proportion

Ratio is used to make a comparison between 2 or more.
A colon (:) is used to split the parts of the ratio. The colon is said as "to" when reading a ratio

Writing a ratio

© © © ® © © © ® © © © ® © © © ®

In the pattern, we can see there are 3 © followed by an ®

The ratio of © to ® is 3 : 1

The ratio of ® to © is 1: 3

The order of the numbers in the ratio matches the order of the words in the sentence

The symbols don't need to be in order to write as a ratio

© © © © ® ® © © ® ® ® ® ®

Above, we can see there are 6 © and 7 ®.

The ratio of © to ® is 6 : 7

The ratio of ® to © is 7: 6

Simplifying Ratios

Ratios can be simplified in much the same way as fractions, by dividing each part of the ratio by the same number

Example 1

© © © ® ® ® ® © © © ® ® ® ®

The ratio of © to ® is 6 : 8

But 6 and 8 can both be divided by 2

So the ratio is simplified as **3: 4**

Example 2

© © ® ® ® © © © © © © © © © ©

The ratio of © to ® is 12 : 3

But 12 and 3 can both be divided by 3

So the ratio is simplified as **4:1**

Splitting Ratio

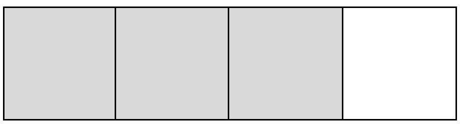
We can split amounts in a given ratio. This can be helpful when mixing liquids or splitting money.

Example 1: Split £120 in the ratio 3:1

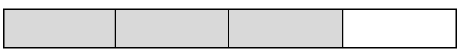
Example 2: Split £84 in the ratio 2:5

Method 1: Bar Model

1) We use a bar to visualise the problem.
   There are 3 blocks to represent the first part and 1 block for the second part – 4 in total.



There are 4 blocks, so each block must be worth £30



£30	£30	£30	£30

In total we can see the ratio is **£90 : £30**

2) There are 2 blocks to represent the first part and 5 block for the second part – 7 in total.

Therefore, each block must cost £12

£12	£12	£12	£12	£12	£12	£12

In total we can see the ratio is £24: £60

Method 2: Calculating

We add up the total number of parts, divide the amount by the total, and multiply each part of the ratio by our calculation

1) 3 + 1 = 4
   £120 ÷ 4 = £30
   3 : 1 = **£90 : £30**

2) 2 + 5 = 7
   £84 ÷ 4 = £12
   2 : 5 = **£24 : £60**

Proportion

Two quantities are proportional if when one doubles, so does the other

Example 1: A car factory produces 1500 cars in 30 days. How many could they produce in 90 days?

Days	Cars
30	1500
90	?

We can multiply 30 by 3 to get to 90. We must do the same to the number of cars

Days	Cars

30	1500
x3	x3
90	**4500**

Example 2: 5 adults buy tickets for the cinema for £27.50. How much do 8 tickets cost?

Tickets	Price
5	£27.50
8	?

We cannot get from 5 to 8 in one simple move, so its easier to calculate the cost of 1 ticket first

Tickets	Price
5	£27.50
÷5	÷5
1	£5.50
x8	x8
8	**£44**

8. Negative Numbers

A number is negative if it is less than 0. It is represented by the – sign in front of the number. It can be shown to the left of 0 on the number line.

Negative direction ← → Positive direction

-9	-8	-7	-6	-5	**-4**	-3	-2	-1	0	1	2	3	4	5	6

The further to the right a number is, the greater the number is.

5 > 3 (5 is greater than 3)

3 > 0

2 > -1

-2 > -5

-4 > -9

Adding and Subtracting with Negative Numbers

Example 1: -3 + 7 Example 2: -4 – 5 Example 3: 6 + -2

<u>Method 1:</u> Rearranging

We can often rearrange or rewrite a problem into a question more familiar to us.

1) -3 + 7 = 7 – 3 (as we can swap the numbers as long as we swap the signs too)

7 – 3 = **4**

2) -4 – 5 = **-** (4 + 5) (both numbers are negative so we can factorise out the negative 1)
-(4 + 5) = **-9**

3) 6 + -2 = 6 – 2 (adding a negative is the same as subtraction)
6 – 2 = **4**

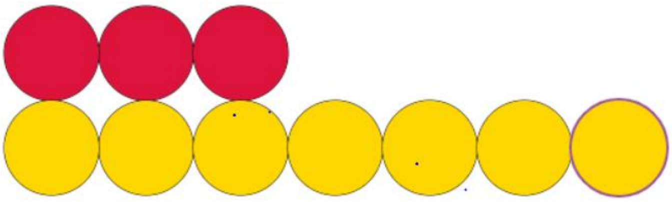
Method 2: Directed Number Counters

We cn use counters to visualise the problem.

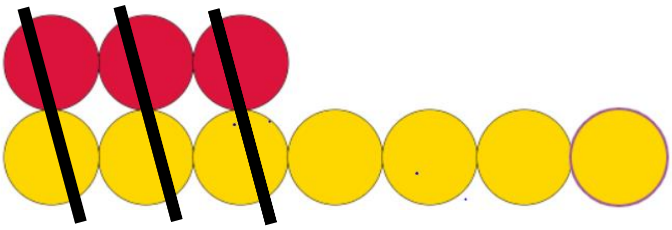
A yellow counter is worth +1 and a red counter is worth -1

1) We begin with 3 red counters to represent -3

We then add 7 yellow counters below since we are adding 7



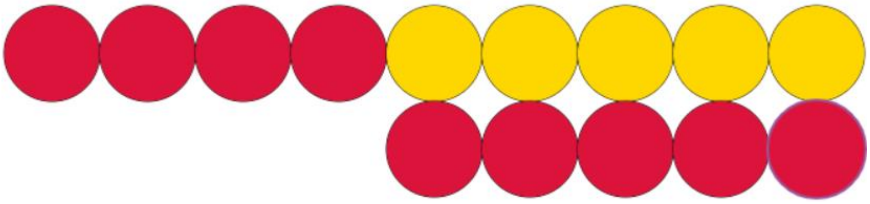
Every red and yellow pair make 0, so we cross any pairs. What remains is our answer.



There are 4 yellow counters left over, so the answer is **4.**

2) We start with 4 red counters to represent -4.

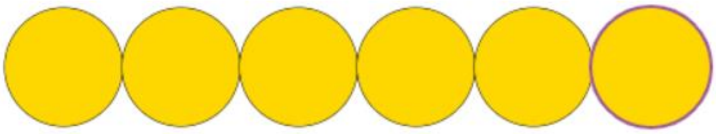
We want to take away 5. We need to remove 5 yellow counters from the picture.
But there are **no** yellow counters
We can add 5 yellow counters if we add 5 red counters too!



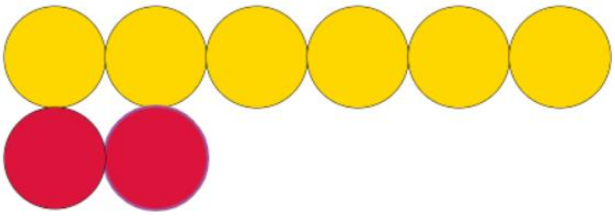
We can now remove the 5 yellow counters, we are left with 9 red counters

The answer is **-9**

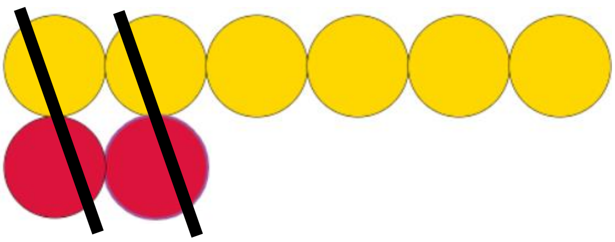
3) Begin with 6 yellow counters to represent +6



Add 2 red counters underneath to represent adding -2



We can cross out 2 red-yellow pair since they meet 0



We can see there are 4 yellow counters left over. The answer is **4**

If you wish to use counters at home, click here to access a free online version of the counters

Multiplying and Dividing Negative Numbers

When multiplying or dividing with negative numbers, we multiply and divide in the same way and think about the signs after the calculations

When multiplying 2 numbers together:

- If the signs are the same, the answer is positive
- If the signs are different, the answer is negative

Examples

-3 x 5 = -15 6 x -4 = -24 7 x -2 = -14

4 x 8 = 32 -2 x -5 = 10 -6 x -2 = 12

-2 x 5 x -3 = -10 x -3 = 30 6 x 3 x -2 = 18 x -2 = -36

18 ÷ -3 = -6 24 ÷ -8 = -3 -36 ÷ 9 = -4

-40 ÷ -10 = 4 -50 ÷ -5 = 10 -32 ÷ -8 = 4

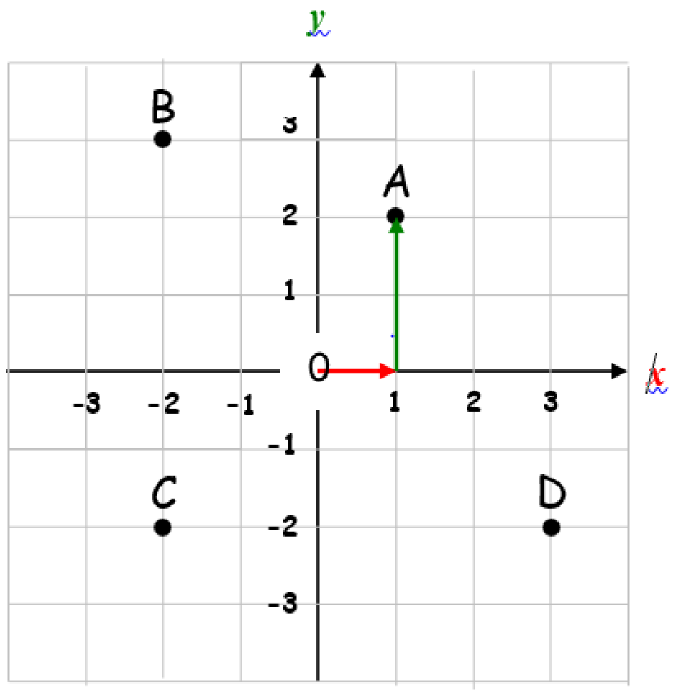
9. Coordinates

Coordinates describe the location of a point in 2 dimensions. There are 2 parts to a coordinate

$$(x\ ,\ y)$$

The x coordinate describes <u>horizontal</u> distance from the origin.
The y coordinate describes <u>vertical</u> distance from the origin

The origin is the starting point. It can be found in the centre of a 4 quadrant grid.



A is the point (1, 2). We start from the origin and move **1** place to the right and **2** places up.

B is the point (-2, 3). We start from the origin and move **2** places to the left and **3** places up

C is the point (-2, -2). We start from the from the origin and move **2** places left and **2** places down

D is the point (3, -2). We start from the origin and move **3** places to the right and **2** places down

10. Inequalities

Inequalities are used to show that one quantity is greater or smaller than the other quantity. There are 4 signs:

Less than	$<$	Greater than	$>$
Less than or equal to	$\leq$	Greater than or equal to	$\geq$

Examples

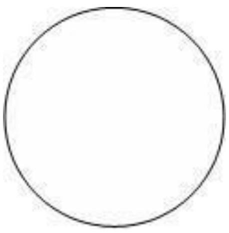
$4 < 8$ (read 4 is less than 8)

$9 > 7$ (read 9 is greater than 7)

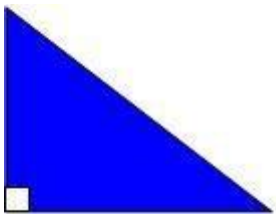
$5 \geq 5$ (read 5 is greater than or equal to 5)

11. 2D Shapes

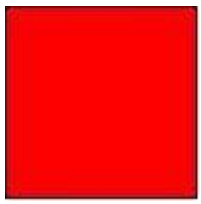
A 2D shape is a flat shape – its 2 dimensions are width and height.



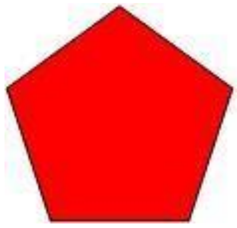
Circle
(1 side)



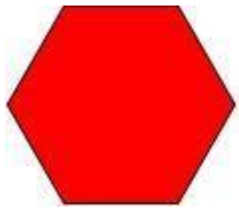
Triangle
(3 sides)



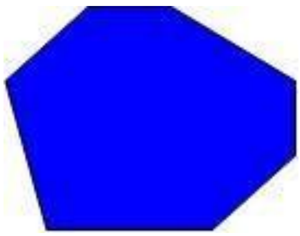
Quadrilateral
(4 sides)



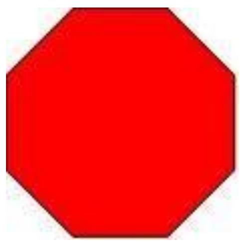
Pentagon
(5 sides)



Hexagon
(6 sides)



Heptagon
(7 sides)



Octagon
(8 Sides)

Polygons

A 2D shape is a polygon if all of its edges are straight lines.

All of the shapes above, beside the circle, are polygons.

A polygon can be either regular or irregular

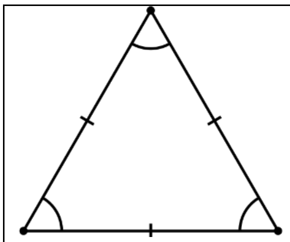
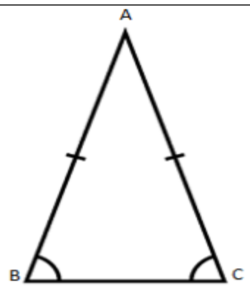
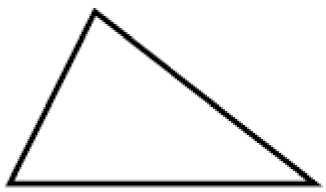
Regular	Irregular
All the sides and angles are the same size	The sides and angles are different sizes

Triangle

A triangle has 3 sides and 3 angles. A triangle is a polygon

The angles in a triangle add up to 180°

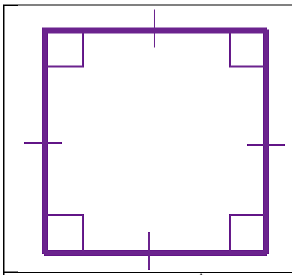
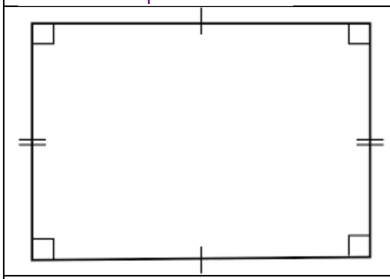
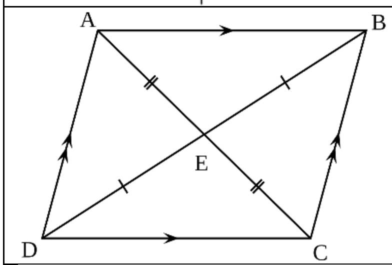
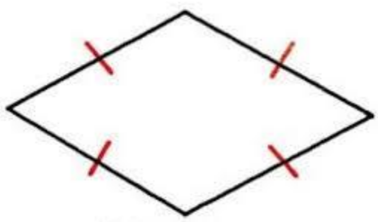
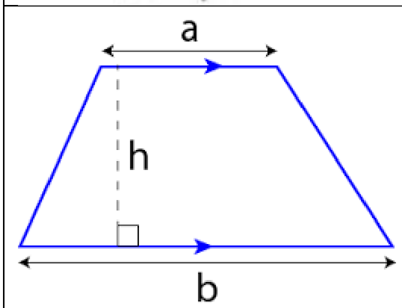
There are 3 types of triangles

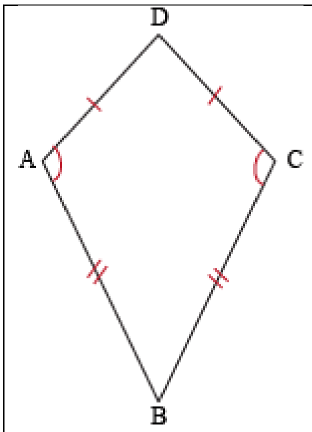
	Equilateral	All sides are equal	All angles are equal
	Isosceles	2 sides are equal length, 1 is different	2 angles are equal size, 1 is different
	Scalene	All sides are different lengths	All angles are different sizes

Quadrilaterals

Quadrilaterals are 4 sides shapes. They are polygons

The angles in a quadrilateral add up to 360°

	Square	All 4 sides are equal length	All angles are right-angles
	Rectangle	Opposite sides are equal length	All angles are right-angles
	Parallelogram	Opposite sides are equal length	Opposite angles are equal size
	Rhombus	All 4 sides are equal length	Opposite angles are equal size
	Trapezium	Sides can vary in length. One pair of parallel lines	Angles can vary in size

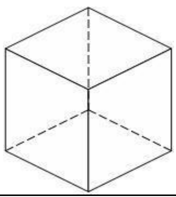
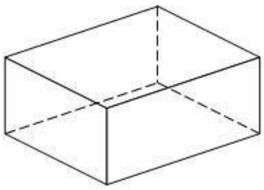
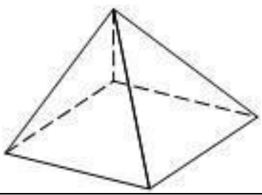
	Kite	Adjacent pairs are equal in length. A short pair and a long pair	I pair of opposite angles are equal

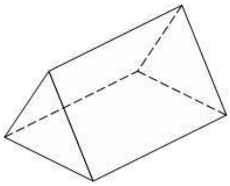
12. 3D Shapes

A shape is 3D because it has 3 dimensions.

We characterise a 3D shape by its faces, edges and vertices.

Face – 2D shape which make the sides of the shape
Edge – Straight line formed where faces meet
Vertex – A point where edges meet.

Shape	Name	Faces	Edges	Vertices (corners)
	Cube	6	12	8
	Cuboid	6	12	8
	Square based pyramid	5	8	5

	Triangular prism	5	9	6

Prisms – A 3D shape made with 2 congruent shapes, connected with rectangles

13. Perimeter

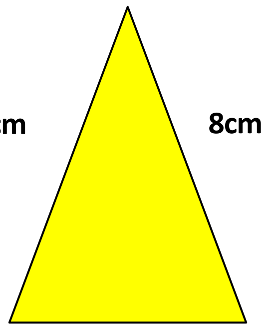
Perimeter is the distance around the outside of a shape. It is measured in standard units – millimetres, centimetres, metres etc.

We can add the lengths of the sides of the shape to calculate the perimeter.

Tip: When calculating perimeter, you add up as many numbers as there are sides. E.g. for a pentagon, you would ass 5 numbers since the shape has 5 sides

Example 1

8cm **8cm**



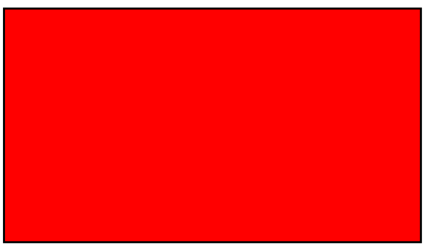
4cm

The triangle has 3 sides, we add all 3 numbers.

8cm + 8cm + 4cm = **20cm**

Example 2 – Missing Numbers

12 cm



5 cm

The rectangle has 4 sides but only 2 measurements. We need to write in the other measurements before we can calculate the perimeter.

Since it is a rectangle, the missing sides are 12 cm and 5 cm

Perimeter = 12cm + 5cm + 12cm + 5cm
= **34cm**

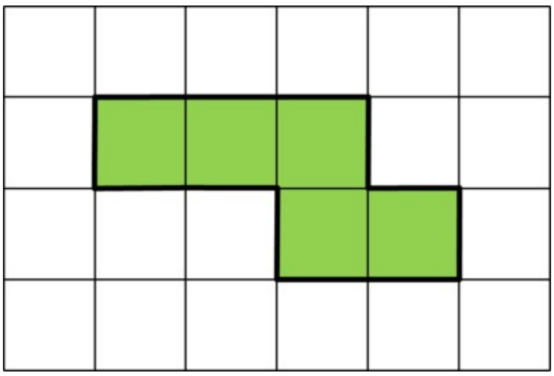
14. Area

This is the space a 2D shape takes up. It is measured in square units:

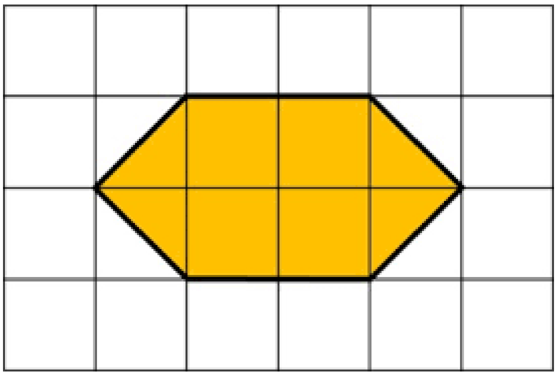
- centimetres squared, cm^2
- metres squared, m^2
- millimetres squared, mm^2

Counting Squares

We can calculate the area of a shape by placing a cm grid over the top and counting the squares. Each square on the grid would be $1cm^2$.



The green shape covers 5 squares.
Therefore the area is **$5cm^2$**



The orange shape has 4 full squares and 4 half squares shaded.

The 4 half squares are equal to 2 full squares

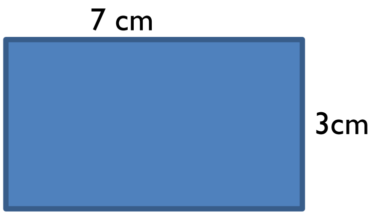
The area is 4 + 2 = **6cm²**

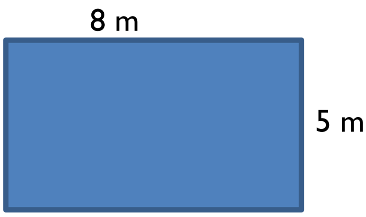
Calculating Area

Area can be calculated by multiplying 2 dimensions together. This sometimes needs to be adjusted through dividing by 2.

Rectangle

Area = Base x Height



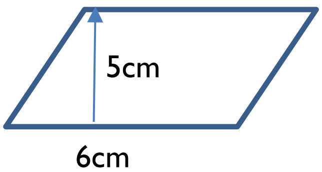


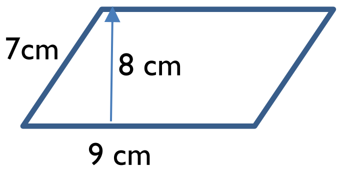
Area = 7cm x 3cm
= **21cm²**

Area = 8m x 5m
Area = **40m²**

Parallelogram

Area = Base x Perpendicular Height





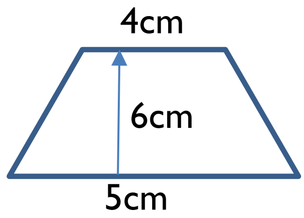
Area = 5cm x 6cm

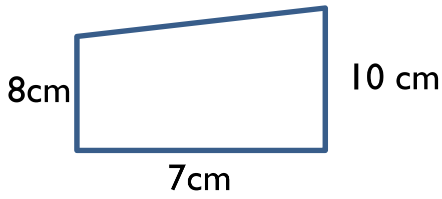
Area = 8cm x 9cm

= 30 cm^2 = 72 cm^2

Trapezium

Area = $\frac{1}{2}$ x Height x Sum of 2 parallel Sides





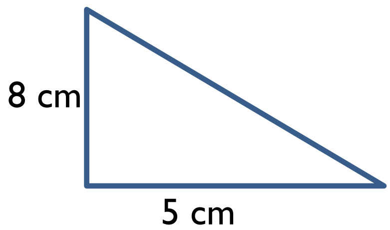
Area = $\frac{1}{2}$ x 6 x (4 + 5)
Area = $\frac{1}{2}$ x 6 x 9
Area = **27 cm^2**

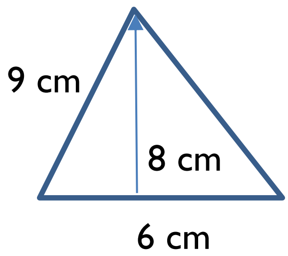
Area = $\frac{1}{2}$ x 7 x (8 + 10)
Area = $\frac{1}{2}$ x 7 x 18
Area = **63 cm^2**

The height is the perpendicular distance between the 2 parallel sides

Triangle

Area = $\frac{1}{2}$ x base x perpendicular height





Area = $\frac{1}{2}$ x 5 x 8
Area = 20 cm^2

Area = $\frac{1}{2}$ x 6 x 8
Area = 24 cm^2

15. Volume

Volume is the amount of space a 3D object contains. It could contain a solid, liquid or gas.

Volume is measured in cubic measurements:

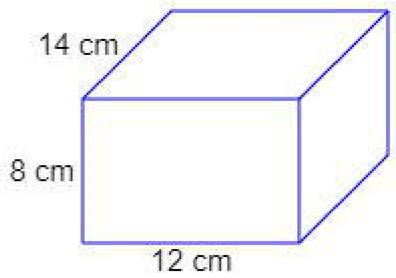
- Cubic millimetres (mm^3)
- Cubic centimetres (cm^3)
- Cubic metres (m^3)

Calculating Volume

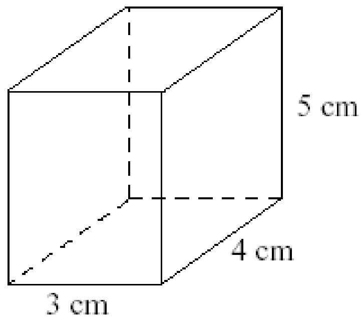
We calculate volume by multiplying 3 dimensions, then correcting by halving when necessary

Cuboid

Volume = Length x Width x Height

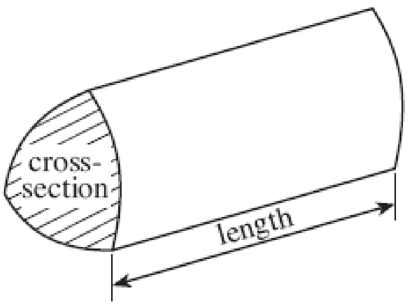


Volume = 8 x 12 x 14
Volume = **1344 cm^3**



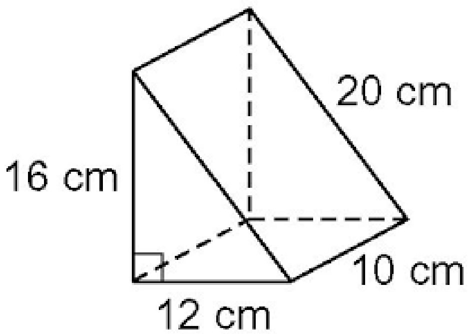
Volume = 3 x 4 x 5
Volume = **60 cm^3**

<u>Prism</u>

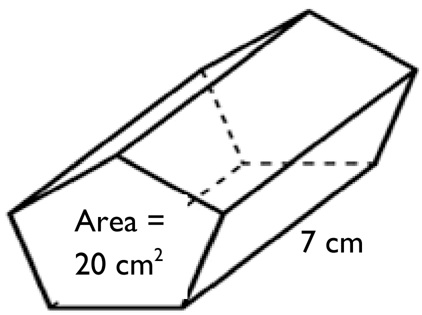


A prism has a consistent cross-section.

Volume = Area of Cross – Section x length



Volume = $(\frac{1}{2}$ x 12 x 16) x 10
Volume = 96 x 10
Volume = 960 cm^3



Volume = 20 cm^2 x 7cm
Volume = 140 cm^3

16. Units of Measurement

Measurement compare the sizes of lengths, mass and volumes. For each, there are 2 types of measurement: metric and imperial.

Length

Metric			Imperial		
Millimetre	mm		Inch	in	
Centimetre	cm	1 cm = 10mm	Foot	ft	1ft = 12 in
Metre	m	1m = 100 cm = 1000 mm	Yard	yd	1 yd = 3ft = 36 in
Kilometre	km	1km = 1000m	Mile		1 mile = 1760 yds

Mass

Metric			Imperial		
Milligram	mg		Ounce	Oz	
Gram	g	1 g = 1000 mg	Pound	Lbs	1 lb = 16oz
Kilogram	kg	1 kg = 1000g	Stone	St	1 st = 14 lb
Tonne	T	1 T = 1000 kg			

Volume

Metric			Imperial		
Millilitre	ml		Pint	pt	
Litre	l	1l = 1000 ml	Gallon	gal	1 gal = 8 pts

Converting Between Metric and Imperial

Length	
1 inch	2.5 cm
1 foot	30 cm
1 mile	1.6 km
5 miles	8 km

Mass	
1 pound	450g
2.2 pounds	2.2 kg

Volume	
1 gallon	4.5 litres
1 pint	0.6 litres
1.75 pints	1 litre

The above conversions are approximate

17. Time

Units of Time

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 week = 7 days

1 year = 12 months = 365 days
1 decade = 10 years
1 century = 10 decades = 100 years
1 millennium = 10 centuries = 1000 years

Clocks

We have 2 types of time clock: 12 hour and 24 hour clocks

The 12 hour clock uses the numbers 1 – 12 for the hours and repeats them twice a day. We us a.m and p.m to differentiate between the two

The 24 hour clock uses the number 0 – 23 for the hours

12 – hour	24 - hour	12 – hour	24 - hour
12.00 a.m.	00:00	12:00 p.m.	12:00
1:00 a.m.	01:00	1:00 p.m.	13:00
2:00 a.m.	02:00	2:00 p.m.	14:00
3:00 a.m.	03:00	3:00 p.m.	15:00
4.00 a.m.	04:00	4:00 p.m.	16:00
5:00 a.m.	05:00	5:00 p.m.	17:00
6:00 a.m.	06:00	6:00 p.m.	18:00
7:00 a.m.	07:00	7:00 p.m.	19:00
8:00 a.m.	08:00	8:00 p.m.	20:00
9:00 a.m.	09:00	9.00 p.m.	21:00
10:00 a.m.	10:00	10.00 p.m.	22:00
11:00 a.m.	11:00	11:00 p.m.	23:00
The morning times are similar regardless of the clock		Add 12 to the hour to convert from 12-hour to 24-hour clock	

Reading the Time

When the number of minutes is 0, we read the time as "o'clock"
When the number of minutes is between 1 – 30, we read the time as "past the hour"
When the number of minutes is between 31-59, we read the time as "to the next hour"

We should never say a bigger than 29 when describing minutes when talking about time

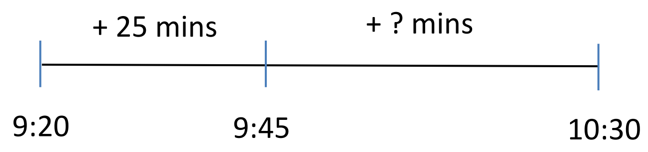
02:10	2:10 a.m.	Ten past two in the morning
07:15	7:15 a.m.	Quarter past seven in the morning
15:20	3:20 p.m.	Twenty past three in the afternoon
21:30	9:30 p.m.	Half past nine in the evening
14:40	2:40 p.m.	Twenty to three in the afternoon
21:45	9:45 p.m.	Quarter to ten at night

Solving Time Problems

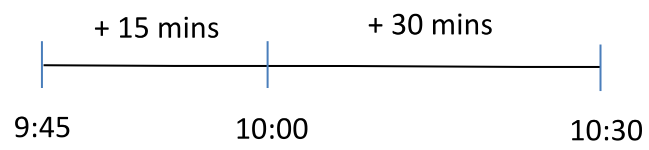
A number line is the most effective way of dealing with problems based around time

<u>Example</u>

I left my house at 9:20. I walked to my friend's house in 25 minutes and then got the bus to school for 10:30. How long did the bus journey take



We need to work out the missing time between 9:45 and 10:30

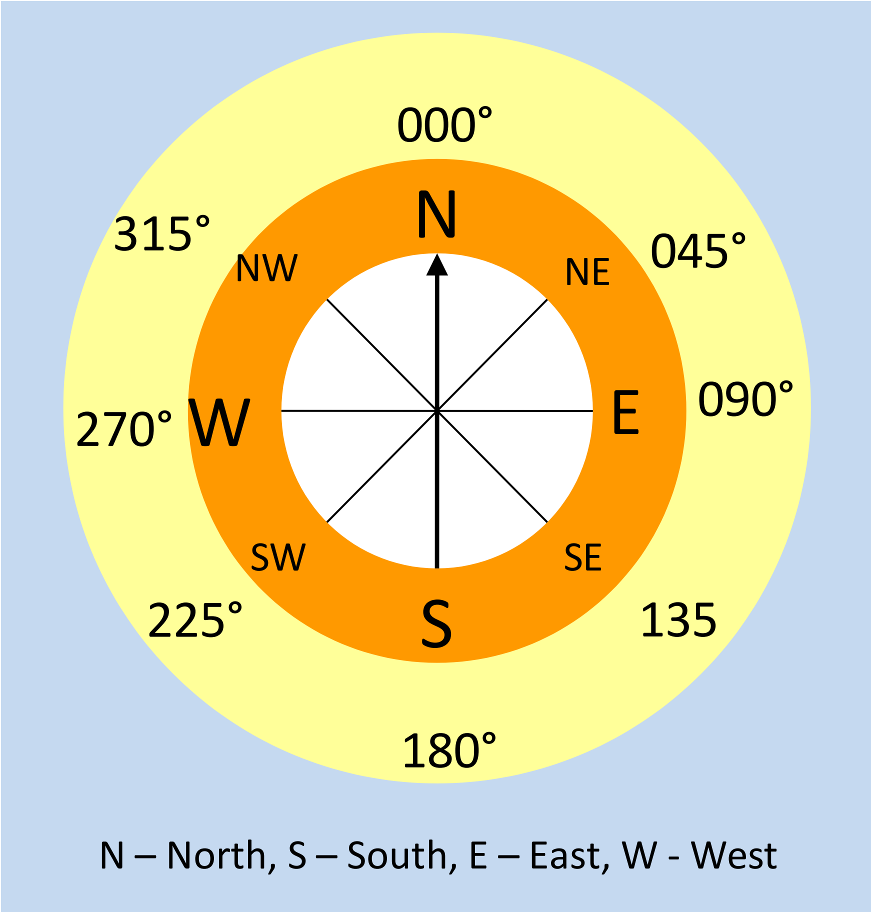


15 + 30 = **45 minutes**

18. Bearings

A bearing describes direction. It tells us how far clockwise from north an object has travelled. Bearings are written as 3 figure numbers. E.g,

030°　　084°　　123°　　321°



19. Displaying Data

There are 2 types of data: Continuous and Discrete

Discrete Data: Data which can only take certain values
e.g. shoe size, favourite colours, number of people in a store

Continuous Data: Data which can take any value on a scale; a level of accuracy is decided on e.g height, weight, distance travelled

Collecting Data

We use tally charts to collect data.

A tally is noted for every appearance of a piece of data. When a there are 5 tallies, the mark is written diagonally to note the fifth. When all tallies are complete, the frequency can be counted.

The data that has been surveyed – these are people's answers

Transport	Tally	Frequency
Walk	卌 卌 |||	13
Bus	卌 ||	7
Car	||||	4
Bike	卌	5
Train	|	1

Each tally is totalled to give the frequency. Counting in 5's makes the process quicker and more accurate

The diagonal line shows we have counted 5.

x mpg	Tally	Frequency
$0 \leq x < 10$	|	1
$10 \leq x < 20$	|||	3
$20 \leq x < 30$	||	2
$30 \leq x < 40$	||||	4
$40 \leq x < 50$	卌	5

This tally chart shows continuous data.

Certain values look like they overlap (such as 10, 20 etc.) but don't through the use of inequality signs.

If a data point of 20 was given, it would go in the 3rd row down ($20 \leq x < 30$) since $x = 20$.

The inequality is sometimes on the right hand side

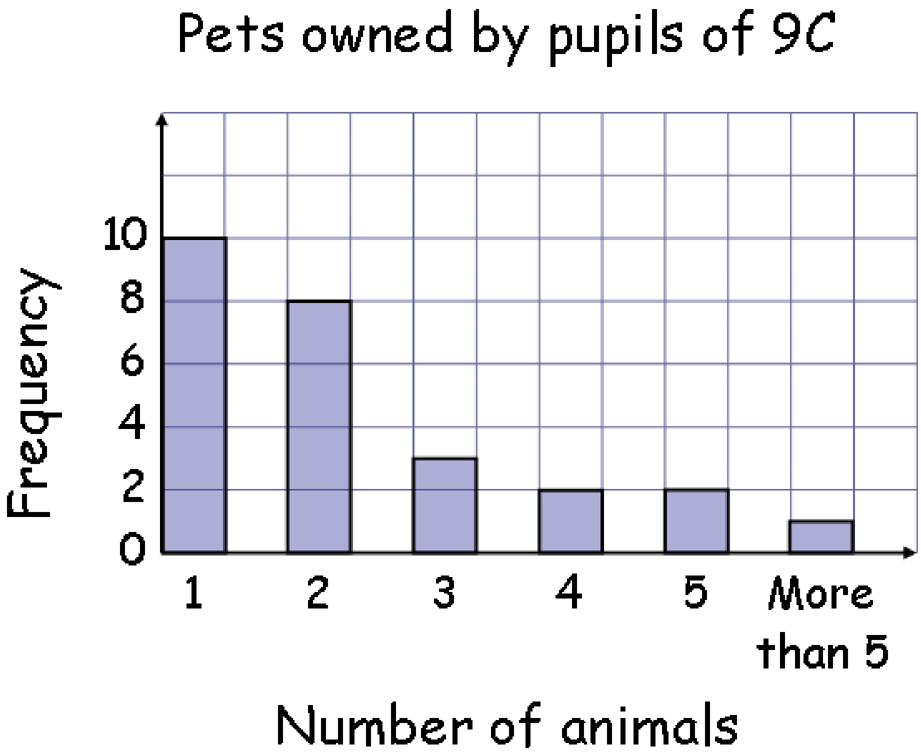
Bar Charts

A bar chart represents discrete data. The bars represent the frequency of each outcome.

All bars must be the same width and there must be a gap between the bars, all gaps are equally sized.

The outcomes (colours, names, score etc.) are noted along the x – axis.

The scale goes up the y – axis. This starts from 0 and goes up in equal increments.

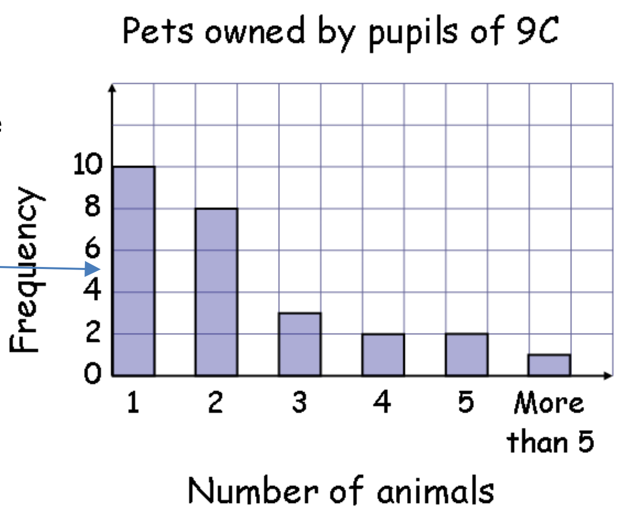


Mode

The mode can be read from a bar chart – it is the

Here, we can see the tallest bar is on the left hand side.

This means the mode is 1 animal



Pie Charts

Pie charts represent discrete data. They show the proportion an outcome has compared to the total.

When constructing a pie chart, you need to calculate the size of the angle each outcome has in the pie chart.

Calculating Angles

The angles fro a pie chart are calculated using the formula

$$Angle = \frac{Frequency}{Total} \times 360$$

When we have a frequency table, we must:

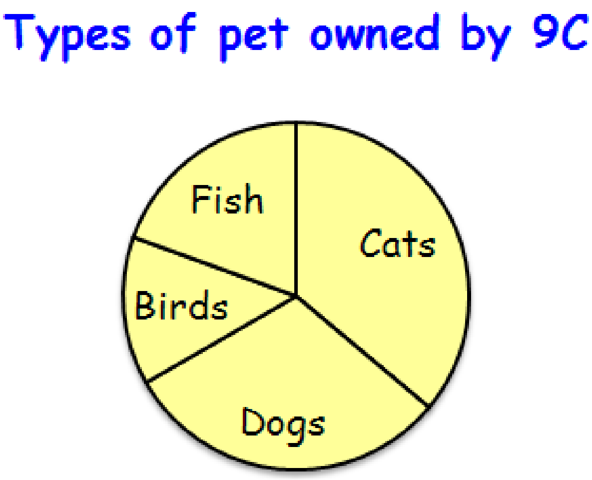
- Add all frequencies to find the total
- Add an extra column to the table
- Perform the calculation for each row of the table in the extra column

Type of pet	Frequency	Angle
Cats	13	$13/36 \ \times 360° = 130°$
Dogs	11	$11/36 \ \times 360° = 110°$
Birds	5	$5/36 \ \times 360° = 50°$
Fish	7	$7/36 \ \times 360° = 70°$
Total	36	360°

To check your answers, add all the angles in the final column. They should equal 360°

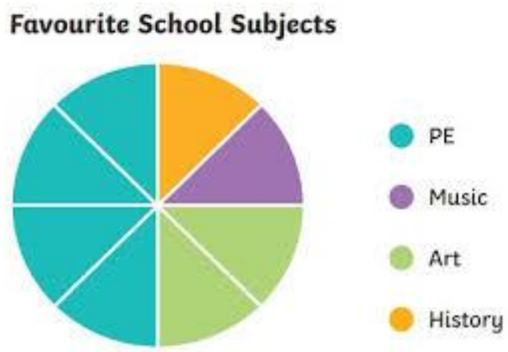
Drawing Pie Charts

To draw a pie chart, you should use a protractor, ruler and pencil. Measure and draw each angle in the table. Once drawn, label the sectors with the appropriate title



Reading Pie Charts

A pie chart tells the reader about proportion rather than the frequency



From this pie chart, we can tell that half the votes were for PE since half of the chart is shaded blue.

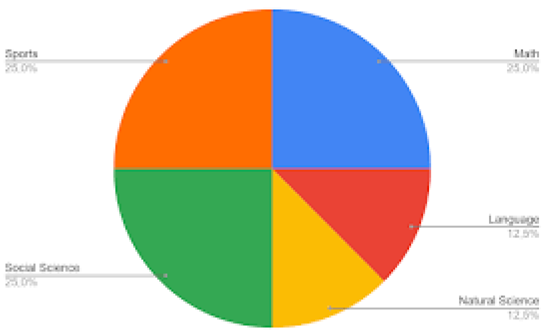
We do not know how many voted for PE since we don't know the total. It could be anything!

Similarly, we know that Music and History got the same number of votes – though not how many they got in total

To calculate the number of votes, we need to know the total and can then use the formula

$$\text{Frequency} = \frac{Angle}{360} \times Total$$

<u>Example</u>



2000 people were asked their favourite subject. How many chose langauges?

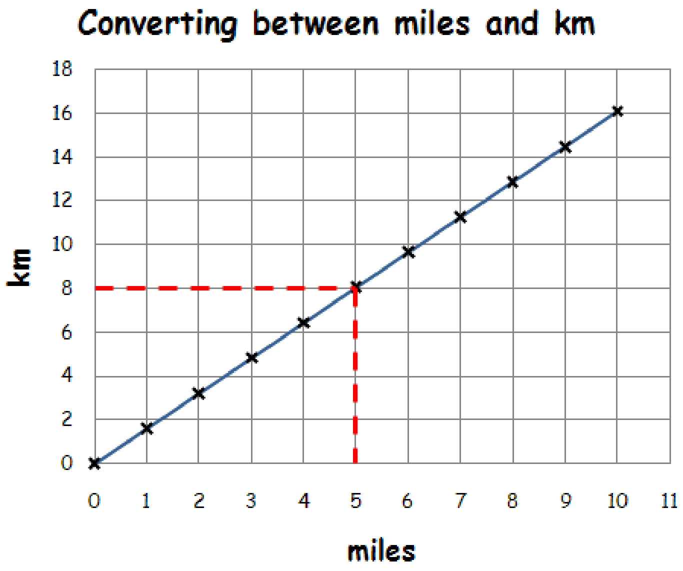
$$\text{Frequency} = \frac{Angle}{360} \times Total$$

$$\text{Languages} = \frac{45}{360} \times 2000$$

$$\text{Languages} = 250$$

Conversion Graphs

Conversion graphs two variables which have a linear relationship. We draw it in the same way as the above graph but the points are connected with a straight line.



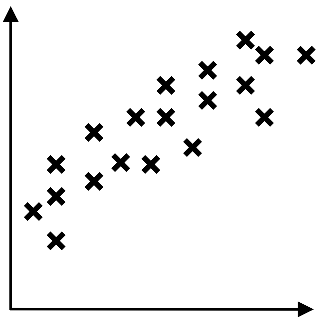
From the graph, we see that 8 km is approximately 5 miles.

Scatter Graphs

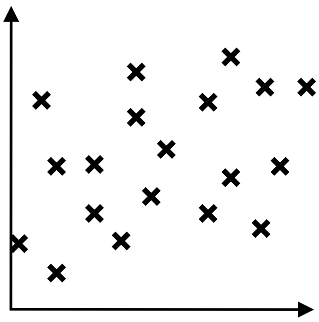
Scatter Graphs show bivariate data – information with 2 variables. We use the scatter graph to compare correlation between the 2 variables.

We might look at rainfall vs. umbrella use, height vs. shoe size, number of pandas vs. languages spoken by locals

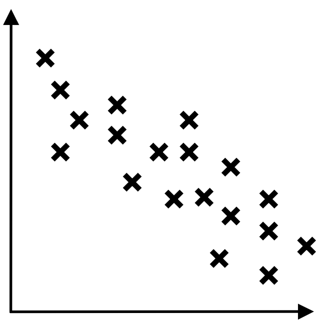
We plot points using a cross but **do not** connect the dots. The dots will fall into patterns called correlation. There are 3 types:



Positive Correlation



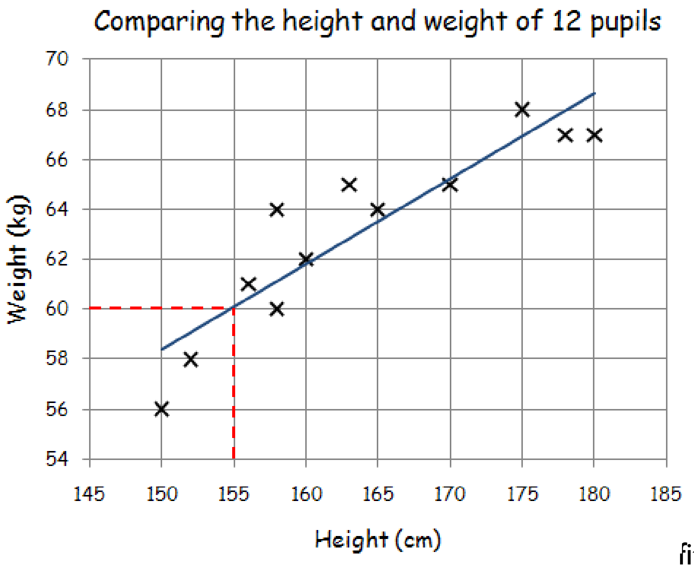
No Correlation



Negative Correlation

If there is a correlation, we can draw a line of best fit on the diagram and use it to estimate the value of one variable given the other.

A line of best fit is drawn by eye – placing a line such that its splits points equally



The line of best fit estimates the relationship between the two variables. Notice that the line follows the trend of the points

There are approximately the same number of points above and below the line.

We estimate that a pupil who is 155cm tall weighs 60 kg

20. Averages

There are 3 measures of average and 1 measure of spread. They are

- Mode
- Median
- Mean
- Range

These can all be calculated from either a list or a frequency table.

Mode

The most common data point in a data set. When looking at a list it is the data point which occurs the most.

Red, Red, Red, Blue, Green, Yellow, Blue. Red occurs the most. **Red is the mode**
3, 3, 4, 4, 5, 5, 5, 6 5 occurs the most. **5 is the modes**

Sometimes data can have multiple modes. If more than one data point has occurred the most times. This is called **bi-modal** data

Red, Red, Blue, Blue, Green, Yellow, Orange
Red and Blue both occur the most. Red and Blue are the mode

4, 4, 4, 5, 6, 7, 7, 7
4 and 7 both occur the most. 4 and 7 are the mode

We can also calculate the mode by reading it from a frequency table. On the table we look for the largest frequency. The outcome with the largest frequency is the mode. It may also be referred to as the modal class.

Colour	Frequency
Red	6
Blue	3
Green	5
Yellow	2
The largest frequency is 6. The mode is **red**	

Goals	Frequency
3	4
4	2
5	6
6	2
The largest frequency is 6. The mode is **5 goals**	

Age,a	Frequency
$0 \leq a < 10$	4
$10 \leq a < 20$	3
$20 \leq a < 30$	3
$30 \leq a < 40$	6
The largest frequency is 6. The **modal class** is $30 \leq a < 40$	

Median

The median is the middle data point in an ordered data set. We must check the list of number is in order before we begin.

Example 1: Find the median of 3, 5, 7, 7, 8, 10, 11

Example 2: Find the median of 5, 8, 9, 11, 15, 20

Method 1 – Crossing out

Cross out the smallest value in the list. Follow this by crossing out the largest value in the list. Continue until you only have a single data point remaining this is the median.

If you have two data points remaining, add them up and divide by 2 to calculate the median.

1) ~~3~~ 5 7 7 8 10 11
~~3~~ 5 7 7 8 10 ~~11~~
~~3~~ ~~5~~ 7 7 8 ~~10~~ ~~11~~
~~3~~ ~~5~~ ~~7~~ 7 ~~8~~ ~~10~~ ~~11~~

7 is the median

2) ~~5~~, 8, 9, 11, 15, 20
~~5~~, 8, 9, 11, 15, ~~20~~
~~5~~, ~~8~~, 9, 11, ~~15~~, ~~20~~
9 + 11 = 20
20 ÷ 2 = 10

10 is the median

Method 2 – Position

We can calculate the position of the median and find that place, rather than crossing out. To do so, we:

- Count the numbers in the list
- Add 1 and divide by 2 – this gives us the position
- Find the number in that position in the list

1) 3 5 7 7 8 10 11
There are 7 numbers in the list
7 + 1 = 8
8 ÷ 2 = 4
In 4th position is the number 7

7 is the median

2) 5 8 9 11 15 20
There are 6 number in the list
6 + 1 = 7
7 ÷ 2 = 3.5
In 3rd position is 9, In 4th position is 11
In the middle is 10

10 is the median

We can also use this method to find the median from a frequency table.

Goals	Frequency
3	2
4	5
5	3
6	5

Total frequency is 15
15 + 1 =16
16 ÷ 2 = 8
Which is the 8th position

If you count up the frequencies, you'll see the 8th data point is a 5

5 is the median

Mean

There are 2 steps to calculating the mean:

- Add up all the numbers
- Divide by the number of numbers

Calculate the mean: 3, 5, 6, 7, 10, 11, 14

3 + 5 + 6 + 7 + 10 + 11 +14 = 56
56 ÷ 7 = **8**

Calculate the mean: -2, 6, 4, 1, 1, 10, 7, -3

-2 + 6 + 4 + 1 + 1 + 10 + 7 + -3 = 24
24 ÷ 8 = **3**

We can calculate the mean from a frequency table using a similar method. When calculating the mean, we must add an extra column and extra row to the table for our calcualtions

- Multiply across (outcome x frequency)
- Sum the frequencies
- Sum the end column
- Divide

Calculate the mean

Goals	Frequency
1	5
2	3
3	2

Goals	**Frequency**
4	3
5	0
6	9

Goals	Frequency	
1	5	1 x 5 = 5
2	3	2 x 3 = 6
3	2	3 x 2 = 6
Sum	10	17

Goals	Frequency	
4	3	4 x 3 = 12
5	0	5 x 0 = 0
6	9	6 x 9 = 54
Sum	12	66

Mean = 17 ÷ 10
Mean = **1.7**

Mean = 66 ÷ 12
Mean = **5.5**

Range

The range is a measure of spread. It shows the consistency of an event.

The range is calculated by subtracting the smallest value from the largest value

Calculate the range: 4, 7, 8, 10, 12, 16

16 – 4 = 12 **The range is 12**

Calculate the range: 4, 1, 9, 5, 10, 3

10 – 1 = 9 **The range is 9**

When we calculate the range from a frequency table, we look at the left hand column (the event).

We subtract the smallest from the largest in this list

Goals	Frequency
2	4
3	5
4	1

Range = 4 – 2
Range = 2

Goals	Frequency
6	5
9	8
11	3

Range = 11 - 6
Range = 5

Glossary

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: 12+76 = 88
a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Data	A collection of information (may include facts, numbers or measurements).
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 50 – 36 = 14
Division (÷)	Sharing a number into equal parts. 24 ☐ 6 = 4
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value.
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than (>)	Is bigger or more than. Example: 10 is greater than 6. 10 > 6
Least	The lowest number in a group (minimum).
Less than (<)	Is smaller or lower than. Example: 15 is less than 21. 15 < 21.
Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers (see p46)
Median	Another type of average - the middle number of an ordered set of data (see p46)

Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.
Mode	Another type of average – the most frequent number or category (see p46)
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (x)	To combine an amount a particular number of times. Example $6 \times 4 = 24$
Negative Number	A number less than zero. Shown by a minus sign. Example -5 is a negative number.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1 ,3 ,5 ,7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done remembered with the acronym BIDMAS.
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a value of 500.
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight).
Prime Number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor.
Product	The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20.
Remainder	The amount left over when dividing a number.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Total	The sum of a group of numbers (found by adding).